

Long-range dependencies in heart rate signals- revisited

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ABSTRACT

The RR series extracted from human electrocardiogram signal (ECG) is considered as a fractal stochastic process. The manifestation of long-range dependencies is the presence of power laws in scale dependent process characteristics. Exponents of these laws: β - describing power spectrum decay, α - responsible for decay of detrended fluctuations or H related to, so-called, roughness of a signal, are known to differentiate hearts of healthy people from hearts with congestive heart failure. There is a strong expectation that resolution spectrum of exponents, so-called, local exponents in place of global exponents allows to study differences between hearts in details. The arguments are given that local exponents obtained in multifractal analysis by the two methods: wavelet transform modulus maxima (WTMM) and multifractal detrended fluctuation analysis (MDFa), allow to recognize the following four stages of the heart: healthy and young, healthy and advance in years, subjects with left ventricle systolic dysfunction (NYHA I–III class) and characterized by severe congestive heart failure (NYHA III–IV class).

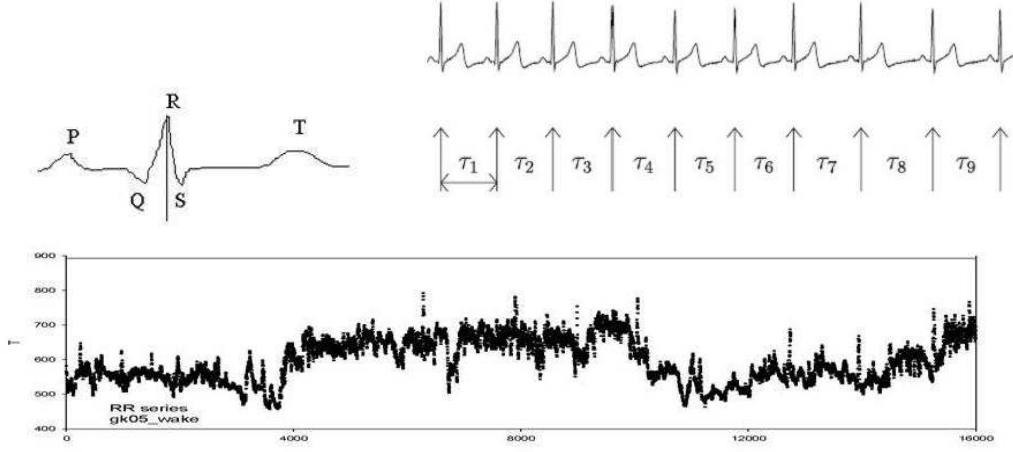


Figure 1: Electrocardiogram analysis: (a) a single P-QRS-T cycle (b) identification of RR-intervals (c) a heartbeat time series from a healthy individual in a daily activity: a normal RR signal.

1 Introduction

Heart rate variability (HRV) represents the most promising marker for measuring activity of the autonomic nervous system — the system that is responsible for cardiovascular regulation in normal and pathologic conditions and is related to mortality [1]. The wide popularity of HRV study is ensured by the noninvasive, easily obtainable techniques providing a signal to analysis [1, 2, 3, 4, 5]. The, so-called, electrocardiogram (ECG), a recording of cardiac-induced potential, reveals the basic information about atrial and ventricular electrical activity of the heart, see Fig 1. Readily recognizable features of ECG are labeled by the letters *P* - *QRS* - *T*. In a continuous ECG record each QRS complex is detected, and the so-called normal-to-normal (NN) intervals between subsequents *R* peaks are determined.

In mathematical terms, the heartbeat signal is modeled as a point process. The occurrence of a contraction at time t_i is represented by an impulse of Dirac delta function $\delta(t - t_i)$, so that the sequence of heartbeats is as follows

$$X(t) = \sum_i \delta(t - t_i) \quad (1)$$

Any realization of a point process is specified by the set of occurrence times

$\{t_i\}$ of these events. The RR-signal $\{\tau_i\}$ is a sequence of positive numbers representing adjacent times $\tau_i = t_{i+1} - t_i$. (The ectopic beats or arrhythmic events which are present in the ECG record and which are not represented in RR series are beside the scope of our investigations, though we remember about their role in quantifying the heart state, see, e.g., [6].)

Clinicians say about normal activity of the heart as regular sinus rhythm. But normal RR-intervals fluctuate in a complex manner. The irregular behavior has motivated researches [7, 8] and still attracts their attention [9, 10, 11, 12, 13] to apply modern tools of statistical physics to uncover hidden dependencies in the series, especially those manifested by long-range power-law correlations.

The well known indicator of the power-law correlation is the β exponent describing decay of the power spectrum $S(f)$. The $S(f) \propto (1/f)^\beta$ scaling with $\beta = 1$ is recognized as manifestation of the fractality of a signal. For many physiological signals, including RR-series $1/f$ -scaling has been found [5, 14]. However, the rigorous meaning of fractality of a signal is related to the Hurst exponent [15, 16]. The Hurst exponent, called also roughness exponent, traditionally measures the statistical dependence of extravagances in a series on the scale a [17]. But the modern definition of H links the roughness with the notion of self-similarity of the process $X(t)$, i.e., with statistical equivalence of the two difference processes: $\{X(t + a\delta) - X(t)\}$ and $\{X(t + \delta) - X(t)\}$, for some $a, \delta > 0$. The equivalence means the probabilistic equivalence of the following measures :

$$X(t + a\delta) - X(t) \equiv_{prob} a^H (X(t + \delta) - X(t)) \quad (2)$$

The Hurst exponent H being the distance dependent value, expresses the so-called long-range dependence embedded in a time series.

Moreover, modern statistical physics provides the method, called detrended fluctuation analysis (DFA), [7] which also detects long-range correlations. The power-law relation between the mean fluctuation of the integrated and detrended subseries and length of the subseries provides the DFA-exponent α . It has been shown that the DFA exponent α differentiates healthy hearts from hearts with a failure [7, 12, 18, 19].

Ubiquity of scale invariant properties emerging from heart rate signals supports the conjecture that RR- series have stochastic rather than deterministic origin since they result from microscopic interactions among many individual components driven by competing forces [4, 14, 8, 12, 20]. To

describe and quantify the mechanisms of these ‘nonhomeostatic’ features, techniques derived from complexity theory: fractal analysis and nonlinear dynamics, are employed. Especially, multifractal analysis provides the resolution of a global exponent to local scaling exponents. It is supposed that a wide range of temporal and spatial scales enable the organism to adapt to the everyday stressful life [20].

At present what it has been found is that the heart rate of healthy humans is a multifractal for which nonzero fractal dimensions and local Hurst exponents are in the interval $h \in (-0.1, 0.5)$ while records for patients with nearly terminal pathology, namely, congestive heart failure, show a significant loss of multifractal complexity by displaying a smaller range of values h [19, 21]. One says that with the heart dysfunction the transition from multifractal to monofractal scaling in the heart variability takes place. Physiologically, this transition is related to the dysfunction of the control mechanisms regulating the heartbeat — the autonomous nervous system.

The autonomous nervous system consists of two parts: sympathetic and parasympathetic which act oppositely to the heart rate. The heart rate is accelerated by the sympathetic system and slowed by the parasympathetic system. Basing on understanding of competing forces gained by critical phenomena study, the origin of heart rate complexity is searched in the intrinsic dynamics of this physiological regulatory system. Fractal scalings in heart rate requires the existence of and intricate balance between antagonistic activity of parasympathetic and sympathetic system. The local multifractal scaling in heart rate effects from the interaction between the activity of sympathetic and parasympathetic nervous systems.

Congestive heart failure is known to be associated with both increased sympathetic system activity and decreased parasympathetic system activity what denotes strong dysfunction of the regulatory system. This disease reveals as a loss of multifractal scalings in the heart rate series. Moreover, if the parasympathetic blocker - atropine, is served to a subject with the healthy heart then a collapse of the multifractal spectrum is observed while heartbeat dynamics during sympathetic blockade displays a small change toward the multifractal spectrum [22]. Furthermore, RR-series of patients with clinically recognized autonomic system dysfunction (loss of sympathetic neurons), though display substantially reduced variability to the levels close to congestive heart failure, the conservation of multifractal properties is observed [11].

The question posed by us is whether subjects with left ventricular systolic

dysfunction (mean LVEF = $30.2 \pm 6.7\%$, NYHA class I–III) is associated with the failure of the autonomic antagonistic activity between parasympathetic and sympathetic nervous systems.

To achieve the goal, we have carefully choose the group of individuals with left ventricular systolic dysfunction and the control group of people with the similar age characteristic and without known any cardiologic history. To both groups the two methods of the multifractal analysis is applied: wavelet transform modulus maxima (WTMM) [23] and multifractal detrended fluctuation analysis (MDFA) [24]. Moreover, we perform the same multifractal analysis to the well-known and widely investigated series from PhysioNet [25] of healthy people, so-called normal sinus rhythm signals, and with near terminal pathology, called congestive heart failure signals. So that we obtain the opportunity to get an insight into the changes of regulatory nervous system caused not only by heart disease but also by advancing in years.

The contents of the paper is organised as follows. In Section 2 the multifractal formalism is introduced. Section 3 contains the description of numerical methods used by us as well as tests for them are presented. The investigations on heart rate series are in Section 4. The results are shown in a series of figures and carefully discussed. The last section, Section 5, concludes our considerations.

2 Multifractal analysis

Multifractal formalism is the way to study scale invariance in their whole variety and dependences [26, 15, 16]. The easily recognizable sign of the scale invariance of a $X(t)$ process is the occurrence of scaling in any of its statistics, called **a structure function**.

For example, let us consider as the structure function the statistics of q -moments of increments of $X(t)$ in a scale a , namely, $\langle |X(t+a) - X(t)|^q \rangle$. If :

$$\langle |X(t+a) - X(t)|^q \rangle \propto |a|^{\tau(q)} \quad (3)$$

then an analysis of scaling phenomena means estimates of the corresponding scaling exponents $\tau(q)$ (called **a partition function**). A stochastic processes exhibiting scaling in their statistics is called a fractal stochastic process.

Depending on relation between τ and q , a fractal process is called:

- **monofractal** — if a partition function $\tau(q)$ is linear: $qH - 1$. Notice,

that the coefficient in this dependence is the Hurst exponent.

- **multifractal** — any other dependence between the partition function τ and the moment q .

On the other hand, the differentiability of any continuous stochastic process can be considered. Roughly speaking, a process $X(t)$ is differentiable at point t_0 if there is a polynomial P_{t_0} , usually the Taylor polynomial of $X(t)$ at t_0 is considered, for which the following relation holds, [16]:

$$|X(t) - P_{t_0}(t)| \propto |t - t_0|^{h(t_0)} \quad \text{for } t \rightarrow t_0 \quad (4)$$

This relation gives rise to the notion of the local Hölder exponent $h(t_0)$ describing the degree of differentiability of a signal at time t_0 . It appears that a typical feature of self-similar processes is that they have a non-integer degree of differentiability, often changing from point to point. Therefore, it is often said that fractal processes are characterized by the inherent, ubiquitous occurrences of irregularities.

Having estimated $h(t)$ for all t , the analysis yields a multifractal spectrum $D(h)$ that may be viewed as a measure of spikiness of the singularity structure of the data. More precisely, if

$$S(h) = \{t : h(t) = h\} \quad (5)$$

is the subset of time interval where the local singularity exponent is equal to h , then the Hausdorff dimension of this set :

$$D(h) = \dim_H S(h) \quad (6)$$

assigns the spectrum value $D(h)$ to the h singularity. The partition function $\tau(q)$ and the singularity spectrum $D(h)$ are mutually related functions by the Legendre transform:

$$D(h) = \min_q (qh - \tau(q)) = q\tau'(q) - \tau(q) \quad (7)$$

$$\tau(q) = \min_h (qh - D(h)) = hD'(h) - D(h) \quad (8)$$

The most prominent example of a fractal process is the fractional Brownian motion $B_H(t)$ [27]. Since at any scale a the following self-similarity relation holds, compare (2) and (3):

$$B_H(at) \equiv_{prob} a^H B_H(t) \quad (9)$$

then all local Hölder $h(t)$ exponents are independent of t and $h(t) = H$ for every t . Hence H , the Hurst exponent, determines differentiability of the Brownian process.

There exists a mathematically rigorous approach to fractional Brownian motions [15, 27, 16] and to their generalizations, to the, so-called random cascades and process of the compound Poisson cascade [28] which explains fractality by multiplicative organization of a signal. Multiplicative cascades are built from iterative split/multiply procedures that hence produce interdependencies between the different scales of the resulting process.

In case of real signals arising from fully developed turbulence [29, 30], financial time series [26, 31] or load of network traffic [32], such local property such as differentiability can be directly related. The Hölder singularity spectrum is accessible by, for example, the wavelet analysis [23].

However, a sequence of events extracted from human heart electrocardiogram is different from signals originally studied by multifractal formalism. The notion of differentiability does not make sense here since the fine scales are not accessible. Therefore by the multifractal formalism one searches for the spectrum of rough singularities, so-called, local Hurst exponents [19, 8, 12].

3 Numerical procedures and tests

Numerical computations of the singularity spectrum for empirical data, straight from the definition, is obviously not feasible. The local singularity exponents vary widely from point to point making their numerical measurement extremely unstable. The way out consists in obtaining the desired multifractal spectrum via carefully designed the structure function. The following two basic structure functions are developed to study nonstationary signals and to determine the partition function:

WTMM — Wavelet Transform Modulus Maxima[23].

Here the multiresolution analysis of a signal is done by the wavelet transform. For a wavelet $\psi(t)$ centered at time zero and scaled a , the wavelet coefficient:

$$W(a, t_0) = \frac{1}{a} \sum_{t=1}^N X(t) \psi\left(\frac{t - t_0}{a}\right) \quad (10)$$

measures the signal content around time $(t - t_0)/a$. The wavelet is chosen orthogonal to the possible trend. If the trend can be represented by polynomial, a good choice for $\psi(t)$ is the m th derivative of a Gaussian $\psi^{(m)} = d^m/dx^m e^{-\frac{x^2}{2}}$. The transform eliminates trends up to $(m - 1)$ -th order. This way the wavelet decomposition contains considerable information on the fractional singularities.

Then as the structure function the average of local maxima of $|W(a, t)|$ as a function of a is chosen:

$$Z(q, a) = \sum_{\substack{\text{local} \\ \text{maxima}}} |W(a, t)|^q \propto a^{\tau(q)} \quad (11)$$

to extract and amplify extravagances in a series.

MDFA — Multifractal Detrended Fluctuation Analysis [24]

Similarly to DFA method the local trend $Y^{(j)}$ for each j -th subinterval of a -th length is determined and the fluctuation for the subinterval is calculated separately:

$$F_j^2(a) = \frac{1}{a} \sum_{k=(a-1)j}^{aj} [Y(k) - Y^{(j)}(k)]^2 \quad (12)$$

Then the structure function means the average over all subintervals of the q th order fluctuation function:

$$F(q, a) = \left\{ \frac{1}{(N/a)} \sum_j [F_j^2(a)]^{q/2} \right\}^{\frac{1}{q}} \propto a^{1 + \frac{\tau(q)}{q}} \quad (13)$$

where $\tau(q)$ has the same meaning as in (11).

For certain values of q the exponents $\tau(q)$ have familiar meaning. In particular, $\tau(2)$ is related to the scaling exponent β of the power spectrum as $\beta = 2 + \tau(2)$ and to the Hurst exponent $H = 1/2(1 + \tau(2))$. For positive q , the structure function describes scaling properties of large fluctuations and strong singularities. For negative q the scaling of small fluctuation and weak singularities is shown. Therefore the partition function $\tau(q)$ by crossing $q = 0$ reveals different aspects of cardiac dynamics [19].

Since in our calculations we use the software accessible from PhysioNet [25], namely, we work with two packets: DFA.C (prepared by J.Mietus, C-K

Peng, and G. Moody) and MULTIFRACTAL.C (prepared by Y. Ashkenazy) we have to perform tests for the methods abilities. The data sets consisting of 20 000 points with known fractal properties are analyzed in two ways: WTMM (with the 3rd derivative of a Gaussian) and MDFA, both in the q -interval $[-10, 10]$ with the step $\Delta q = 0.1$.

The following tests have been performed:

- Multifractality of white noise and random walk, see Fig. 2.

The series are generated by `lrnd48()` function accessible in ANSI C on UNIX workstations. The expected result is the linear partition function $\tau(q)$ with the coefficients 0 for a white noise and 0.5 for a random walk. One can notice a large departure from the theoretical predictions if the partition functions are considered in the large q -interval. Especially, the wrong results are obtained when q is negative and WTMM method is applied. The MDFA method provides correct values in the whole q -interval in case of a white noise, namely, the spectrum is a point : $\{(0, 1)\}$. But in case of a random walk, the point-like spectrum is satisfactory only when the q -interval is shrunk , e.g., to $|q| < 3$.

- Multifractality of a fractional Brownian motion, see Fig. 3.

The series are generated with the help of *tsfBm* packet [33]. From Fig. 3 one can read that the partition functions are not linearly depended on q , especially, when WTMM method with negative q is calculated. Therefore the corresponding spectra are not point-like, they are not symmetrical — they have wide right wings. Sizes of these spectra are related to the method. MDFA method leads to results which systematically diverge from the expected linear behavior as $|q|$ grows. When restricting the q -interval then both methods provides the expected shape, namely, a point spectrum $\{(H, 1)\}$.

- Multifractality of binomial series, Fig. 4, [26]

The binomial measure is the simple example where a strictly multifractal spectrum appears. The binomial series represent a binomial cascade measure. These cascades are constructed through the following multiplicative iteration schema: Let divide the interval into two parts and assign a mass m to the right part and $1 - m$ to the left part of the division. Then, repeat in each

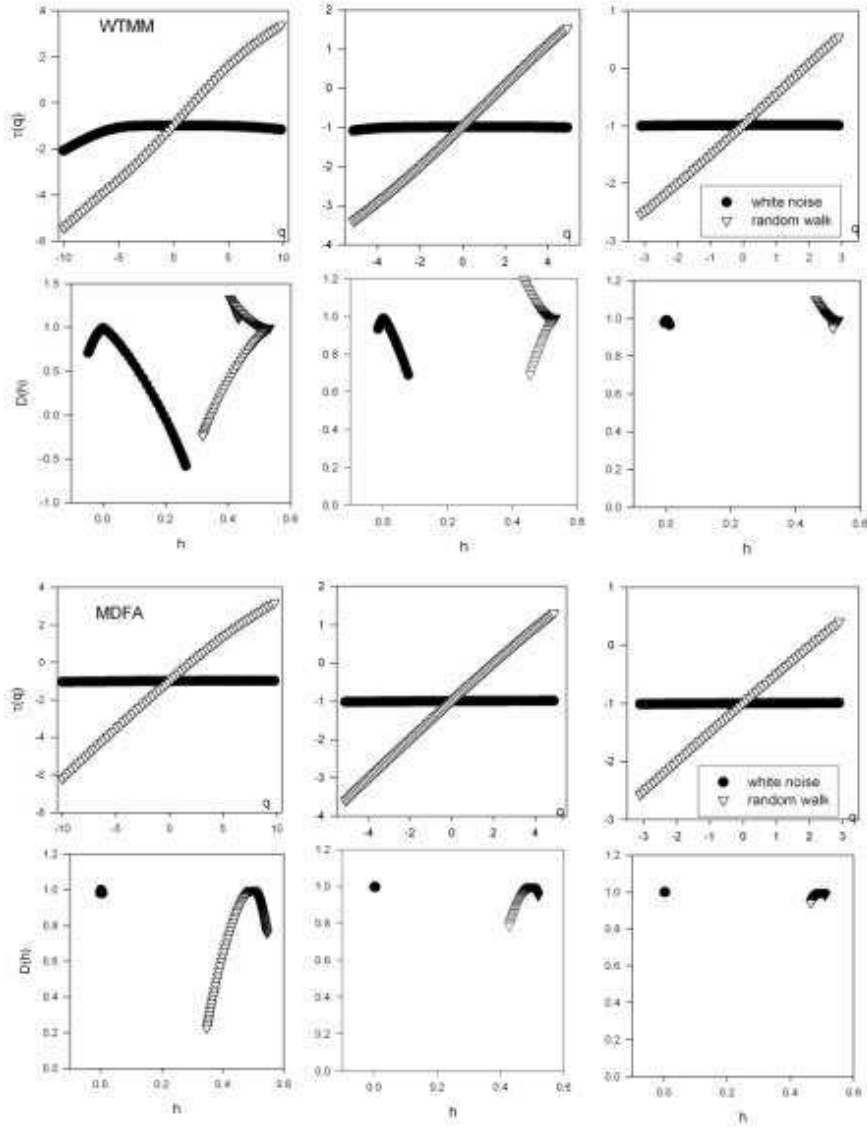


Figure 2: Dependence on q of multifractal spectra in case of a white noise and a random walk for WTMM (top figure) and MDFA (bottom figure). In the subsequent figures the q -interval is decreased.

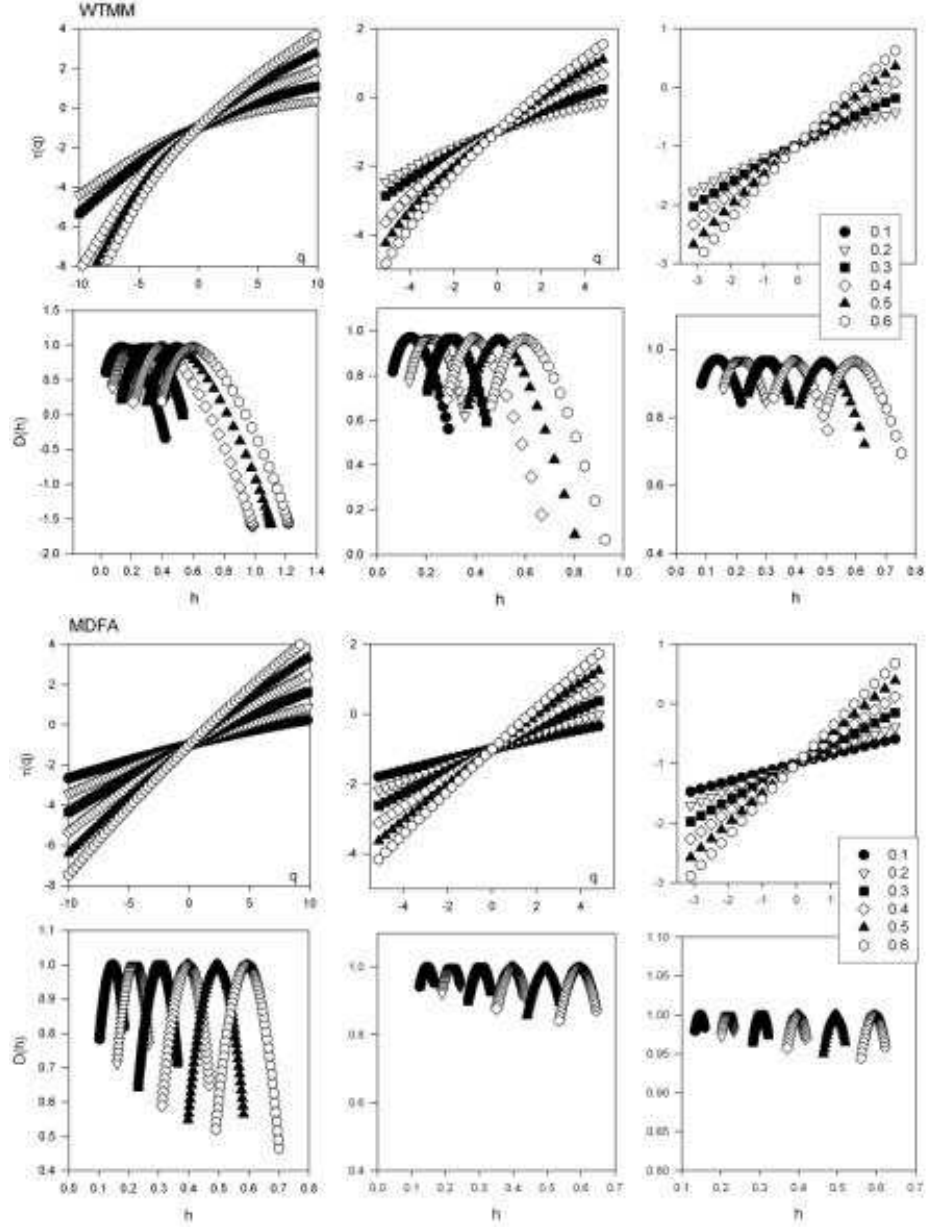


Figure 3: Dependence on q of multifractal spectra in case of fractal Brownian motions if WTMM (top figure) and MDFA (bottom figure) methods are applied. In the subsequent figures the q -interval is decreased. Different point markers correspond to different Hurst exponents as denoted by the labels.

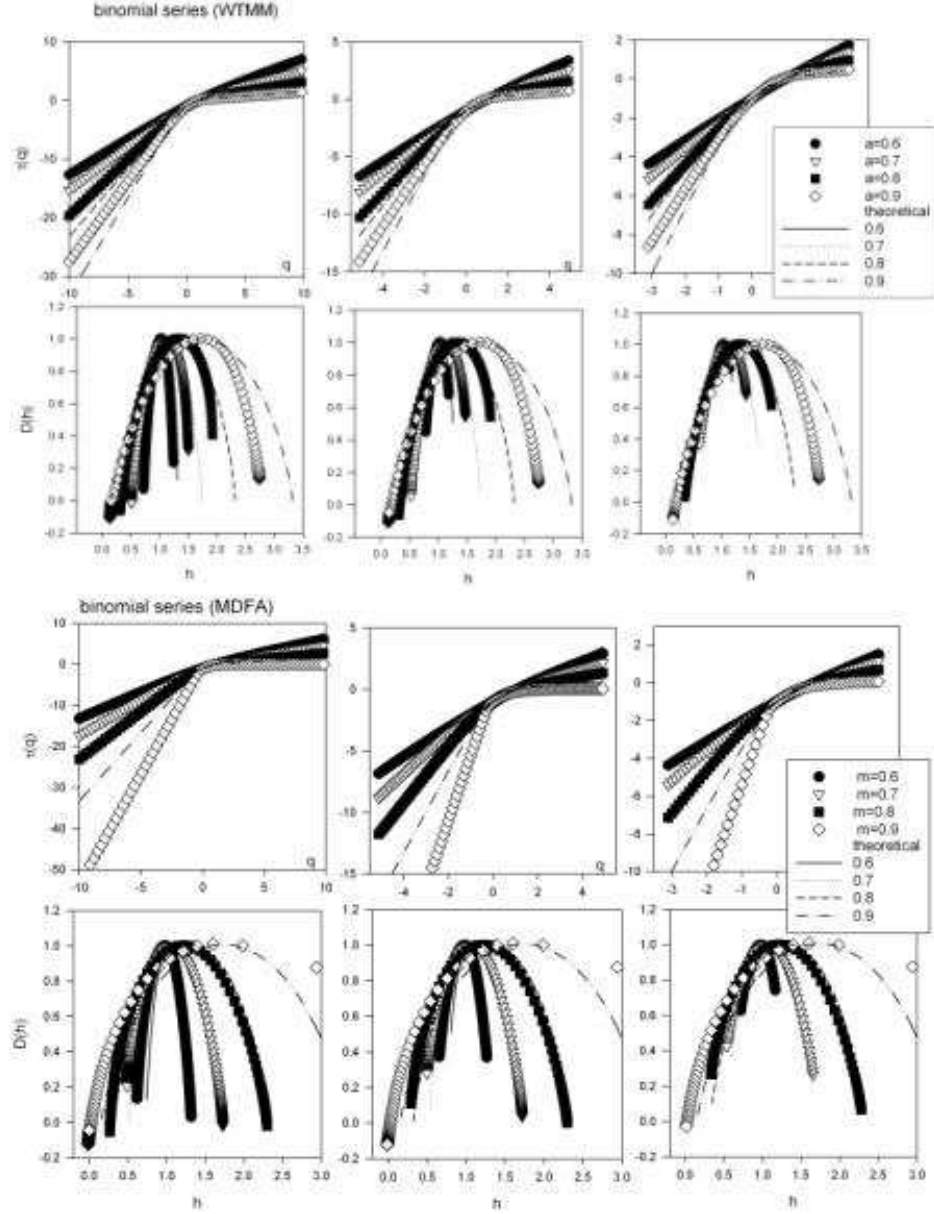


Figure 4: Dependence on q of multifractal spectra in case of binomial measures with different mass values and for WTMM (top figure) and MDFA (bottom figure). In the subsequent figures the q -interval is decreased.

iteration, each of the interval divide into two parts and assign for the right part: the whole interval mass multiplied by m , and for the left part: the whole interval mass multiplied by $1 - m$.

The theoretical approach gives the following values for $\tau(q)$

$$\tau(q) = \frac{-\ln[m^q + (1 - m)^q]}{\ln 2} \quad (14)$$

Both methods WTMM and MDFA give unreliable results if $q < 0$ — WTMM method underestimates the theoretically calculated τ values while MDFA method overestimates these theoretical values what effects in shrinking (WTMM) or enlarging (MDFA) the spectra. On the other hand, for positive q , both methods reveal the expected properties. Noticeable, that independently of the q -interval the large convex spectrum shape is preserved.

4 Heart rate study

The multifractal analysis of RR-series is done for the data downloaded from PhysioNet page [25] and for RR-series collected and selected by us and accessible on request [34]. The 24-hours ECG Holter signals are carefully analyzed and annotated to extract normal RR signals.

4.1 The series

We analyze four groups of subjects:

- **nk** group. The **nk** group (the basic group of the study) consists of 90 patients hospitalized during 2001–2004 years in the 1st Department of Cardiology of Medical University in Gdańsk, Poland (9 women, 81 men, the average age is 57 ± 10) in whom the reduced left ventricular systolic function was recognized by echocardiogram due to the low left ventricular ejection fraction ($\text{LVEF} \leq 0.40\%$, mean $\text{LVEF} = 30.2 \pm 6.7\%$). The additional criteria which exclude subjects from the **nk** group are: the myocardial infarction in last 6 months, persistent atria fibrillation, sinus-node disease, recognized diabetes mellitus, coronary revascularisation in last 6 months or kidney failure with creatine level > 2 .

- **gk** group. The control group is made of 40 healthy individuals (4 women, 36 men, the average age is 52 ± 8) without past history of cardiovascular disease, with both echocardiogram and electrocardiogram in nor-

mal range. The left ventricle ejection fraction was normal (mean LVEF = $68.0 \pm 4.7\%$).

For each person from the above two groups the 24-hour ECG Holter monitoring was performed. The signal was digitized using Delmar Avionics recorder (Digitorder) and then analysed and annotated using Delmar Accu-plus 363 system (fully interactive method) by an experienced physician. The minimum number of qualified sinus beats required for the signal to enter the study is 90%.

— **chf** group. The group consists of 13 patients with congestive heart failure (3 women, 10 men, the average age is 56 ± 12) of whom the whole day data are available from the PhysioNet [25] (www.physionet.org/physiobank/databas/chfdb). This severe heart failure is known to be related to the strong dysfunction of the autonomic nervous system.

— **nsr** group. The group of 14 healthy and rather young people (11 women, 3 men, the average age is 35 ± 8) whose ECG recordings are available from the PhysioNet [25] (www.physionet.org/physiobank/databas/nsrdb).

The PhysioNet database consists of 18 **nsr** records and 15 **chf** signals. From 24-hour series downloaded from the PhysioNet the normal beats were determined following annotations. Only series with the number of normal annotations greater than 80% are included into considerations.

From each signal the two 5-hour continuous subsets were extracted: diurnal and nocturnal. These subsets were extracted manually, following the easy recognizable in RR-series the two stages: daily activity and sleep state. In most cases these periods are consistent with hours 15:00–20:00 for the daily activity and 0:00–5:00 for the sleep state.

Finally, we deal with 8 groups of series named subsequently: **nk_wake**, **gk_wake**, **chf_wake**, **nsr_wake**, **nk_sleep**, **gk_sleep**, **chf_sleep**, **nsr_sleep**. For each series the partition function $\tau(q)$ is calculated by both methods: WTMM and MDFA in the interval $q = -10, \dots, 10$ with a $\Delta q = 0.1$ step. The exponents $\tau(q)$ are obtained from slopes of $Z(q, a)$ by (11) in case of WTMM method and $F(q, a)$ by (13) in case of MDFA method for scales $a > 10$. Then the group averaged partition function is found and the multifractal spectrum is calculated following the Legendre transform (8). The standard deviation errors of the estimated partition functions in case of MDFA do not exceed 5% for all q while in case of WTMM method the errors grow significantly with $|q|$ departing from 0 as it is shown in Fig. 5 in case of **gk_wake** and **nk_wake** RR-series. Similar results are obtained for other series.

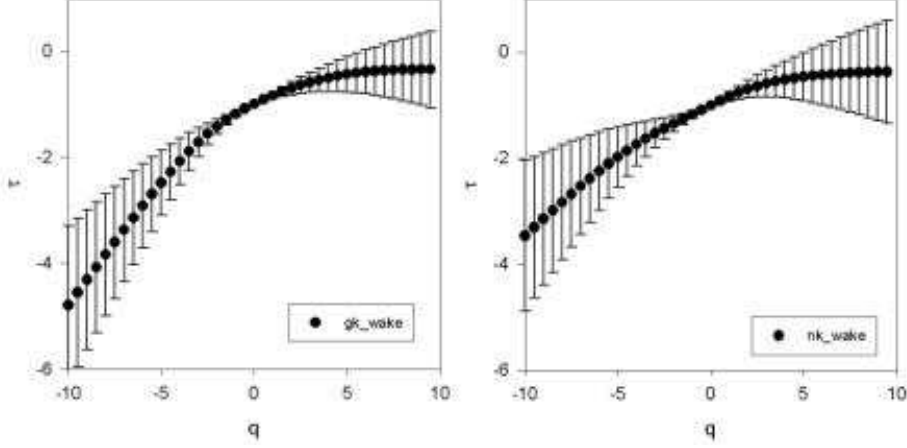


Figure 5: Average partition functions together with standard deviation errors obtained when WTMM method is applied to gk_wake and nk_wake RR-series.

4.2 The results

The results of multifractal analysis are presented in Figs. 6, 7 and in Table 1, 2. We show the results in the whole studied q -interval, but our analysis concentrates on $|q| < 3$ interval to decrease possible numerical effects discussed in the previous section.

If q is positive then both methods provide the **nsr_wake** partition function $\tau_{nsr_wake}(q)$ distinct from others. The resulting spectrum is the widest. However, **gk_wake** and **nk_wake** also lead to the wide spectra but the spectra locations are moved toward higher h values (compare h_l , h_{max} and H in Table 1). The spectrum obtained from **chf_wake** series is different. It is significantly narrow. The presented results allow us to claim that aging and disease influence the heart complexity. The process has two stages. At first, the anticorrelation (feedback) mechanisms are weakened, namely, the response of the autonomic nervous system is reduced, and then, secondly, the autonomic system response is restricted to few regulatory mechanisms only.

Results obtained for the sleeping human state are consistent with the previously observed fact that multifractality is not reduced at minimised physical activity such as constant routine protocol, where physical activity and postural changes are kept to a minimum [22, 11]. The nocturnal activity

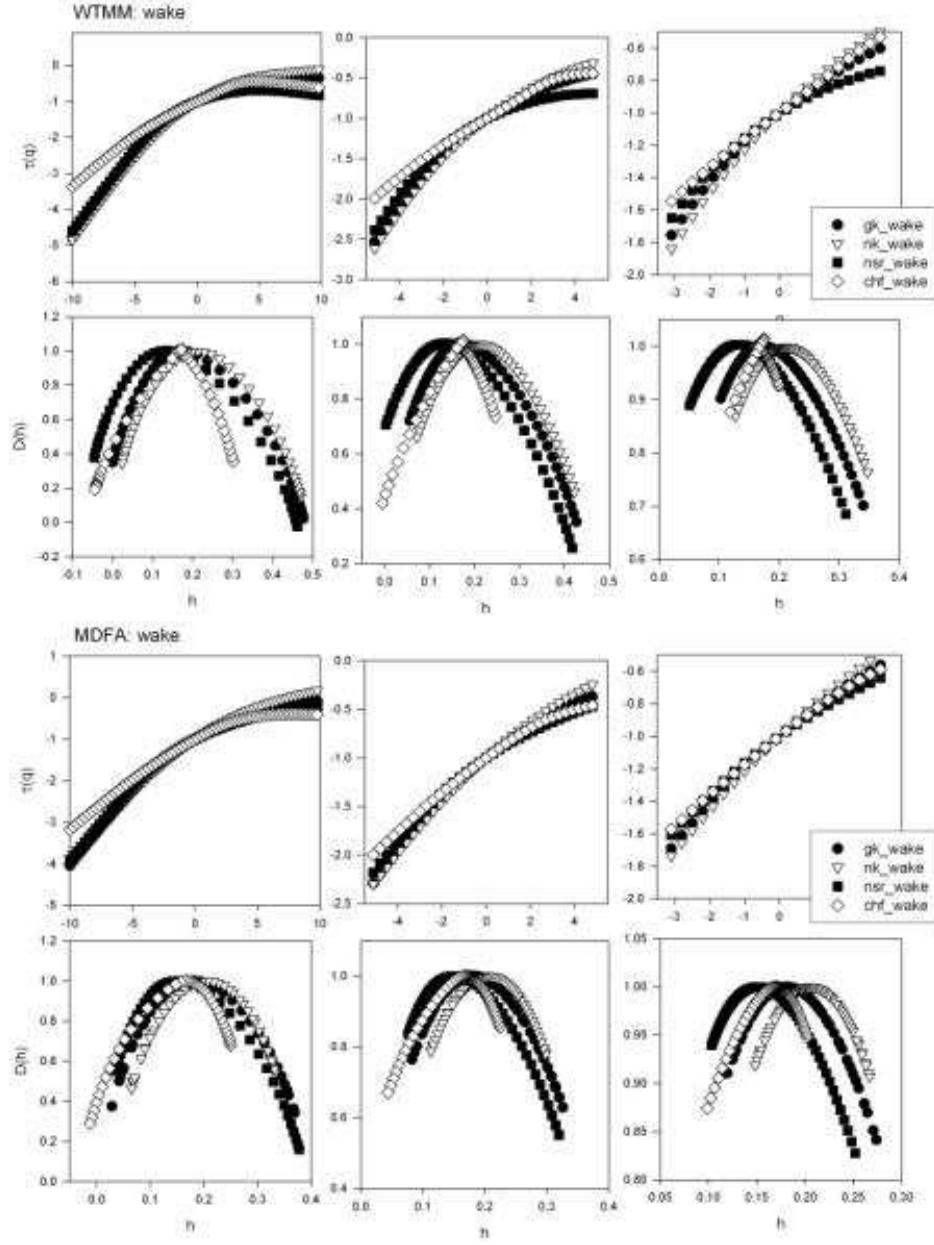


Figure 6: Dependence on q of multifractal spectra for wake series if WTMM (top figure) and MDFA (bottom figure) methods are applied. In the subsequent figures the q -interval is decreased.

Table 1: Properties of multifractal spectra for $|q| < 3$, daytime series. h_l denotes the most left value of the local Hurst exponent noticed in the interval, $D(h_l)$ means its probability; h_r denotes the most right value of the local Hurst exponent noticed in the interval, $D(h_r)$ means its probability; h_{max} is the value of the singularity exponent when q crosses the zero. H is the global Hurst exponent calculated as $H = \frac{1}{2}(1 + \tau(2))$

series	h_l $D(h_l)$	h_r $D(h_r)$	total width	h_{max} Hurst	h_l $D(h_l)$	h_r $D(h_r)$	total width	h_{max} Hurst
	WTMM				MDFA			
gk_wake	0.102 0.90	0.333 0.72	0.231	0.17 0.15	0.120 0.91	0.271 0.85	0.149	0.18 0.16
nk_wake	0.125 0.86	0.341 0.78	0.216	0.21 0.19	0.146 0.91	0.267 0.91	0.121	0.21 0.19
nsr_wake	0.049 0.88	0.304 0.71	0.255	0.13 0.10	0.102 0.94	0.248 0.84	0.146	0.15 0.13
chf_wake	0.111 0.85	0.199 0.93	0.088	0.17 0.17	0.095 0.86	0.200 0.95	0.105	0.17 0.15

Table 2: Properties of multifractal spectra for $|q| < 3$ in case of nighttime series, MDFA method only. Notation is the same as in Table 1.

series	h_l $D(h_l)$	h_r $D(h_r)$	total width	h_{max} Hurst
	MDFA			
gk_sleep	0.013 0.84	0.254 0.82	0.241	0.13 0.08
nk_sleep	0.041 0.86	0.230 0.88	0.189	0.14 0.10
nsr_sleep	-0.004 0.84	0.194 0.87	0.198	0.11 0.06
chf_sleep	-0.001 0.85	0.155 0.92	0.156	0.11 0.07

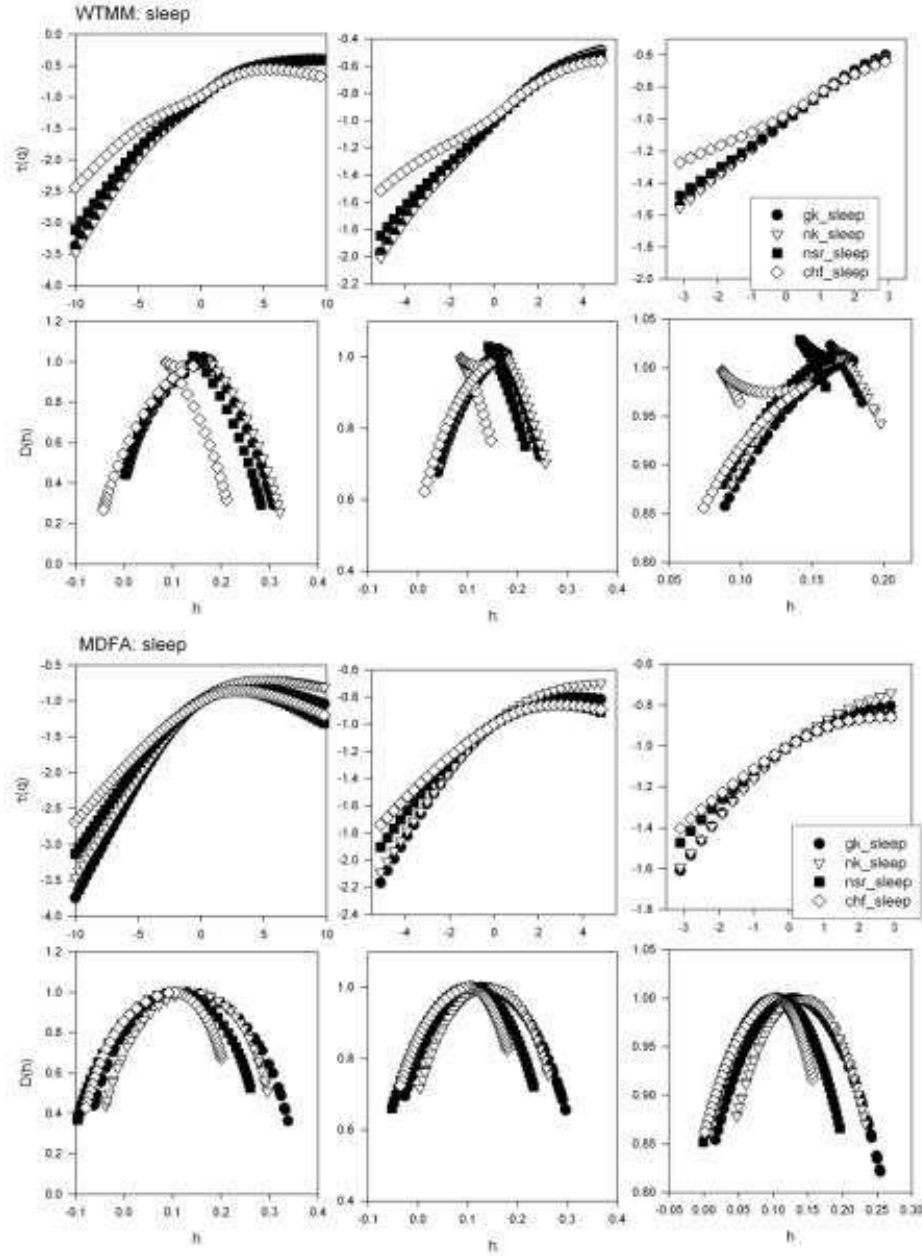


Figure 7: Dependence on q of multifractal spectra for sleep series if WTMM (top figure) and MDFA (bottom figure) are applied. In the subsequent figures the q -interval is decreased.

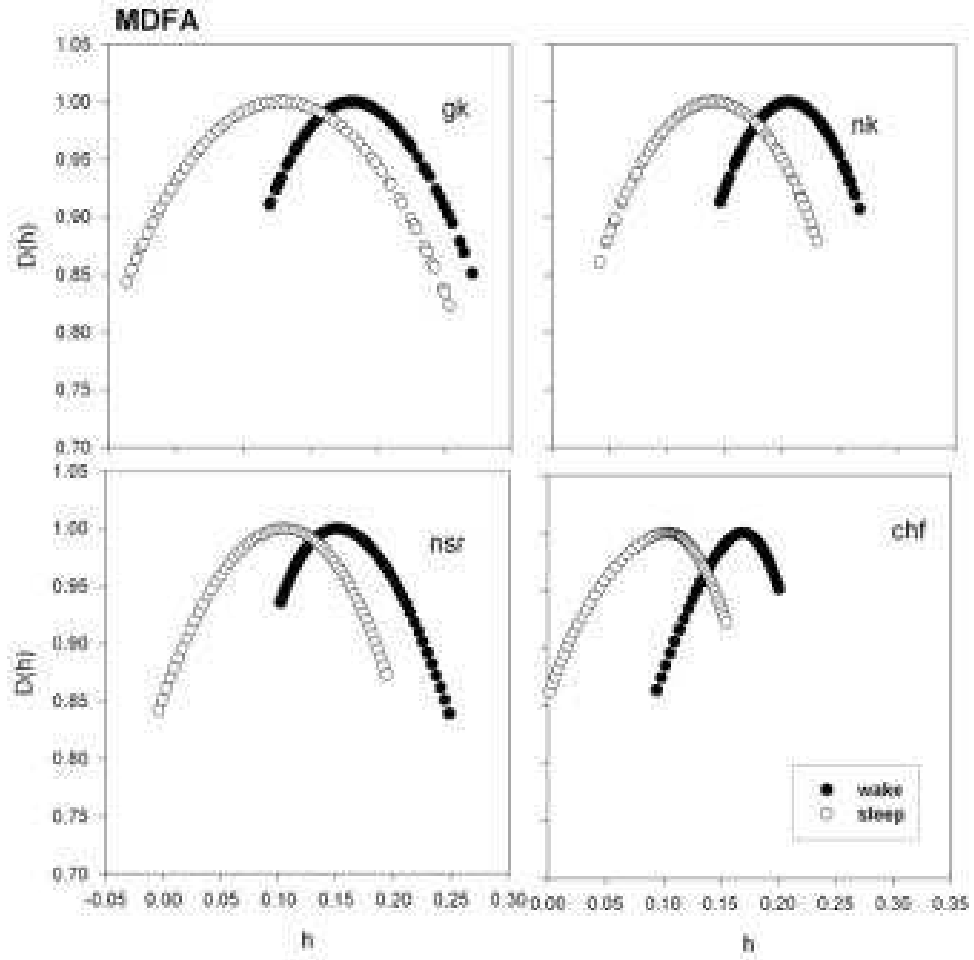


Figure 8: Multifractal spectra for wake and sleep series if MDFA is applied for $|q| < 3$.

does not reduce variability of singularity exponents, also.

The WTMM method leads to non-convex partition functions what effects in numerical instability in estimates of the spectra, though the MDFA method provides regular shapes, see Fig. 7. Therefore we present the multifractal characteristics corresponding to the MDFA method only, see Table 2. To amplify difference between diurnal and nocturnal series we show the MDFA spectra in the subsequent panels of Fig. 8. The nighttime spectra are moved to lower h values but the spectra are wider than the spectra of corresponding wake series. The global Hurst exponent is significantly lower. Hence the results indicate even increase of the impact of the regulatory system at the sleeping state, and rather support the hypothesis that there are changes in cardiac control during sleep and wake periods which lead to the systematic changes in the scaling properties of the heartbeat dynamics, see e.g. [12, 21].

5 Conclusions

The cardiac time series exhibits wild irregular fluctuations and nonstationary behavior in the healthy subject which is much attenuated in cardiac diseases and/or by aging. The question posed by us in this paper whether the left ventricular dysfunction results in the change of multifractal properties.

Both methods: WTMM and MDFA show that the transition in the correlation properties takes place. From strong and multi-scale anticorrelations — reach multifractal state, in case of healthy and young hearts, by weakening these properties (less anticorrelation and less multi-scales) caused by advance in years of a subject, and further loosing control in case of a subject with left ventricle systolic dysfunction. Finally, domination of few scales occur — a monofractal state, when the heart is seriously damaged with clinical manifestation of the congestive heart failure. The spectra of wake series obtained by both methods display these properties: the shrink of h interval when we move from the healthy heart to the heart with a disease and the change in the shape of spectrum from parabola-wide shape to triangle-like shape. The nocturnal data provide different property — the anticorrelated feature of the heartbeat fluctuations increases during the sleep human state.

We believe that our findings are important in putting forward the role of multifractal analysis for diagnose the conditions a range of patients

Acknowledgments

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